Testing the integration rule in intertemporal choice models

Elena Villalobos
National Autonomous University of Mexico

Arturo Bouzas
National Autonomous University of Mexico

Michael D. Lee
University of California Irvine

One of the main questions in the intertemporal choice literature is how we evaluate alternatives that differ in amount of reward and delay to delivery, that is, how we integrate and compare information to choose between an alternative that has a small reward but it is delivered immediately and an alternative that has a larger reward but its delivery is delayed in time. There are two great model families that describe these kinds of choices that have a different integration rule, one is the alternative-based family and the other is the attribute-based family. Interval effects are empirical phenomena that distinguish between both families. The objective of the present work was to study interval effects and to compare, by applying Bayesian statistical tools, models that can describe these phenomena. Results showed that some subjects followed an alternative-based rule while others an attribute-based one. Among the attribute-based family, the Direct Differences model was the simplest and best one to describe the present data set.

Daily, organisms face choice situations among outcomes that involve tradeoffs occurring at different points in time, which are named Intertemporal Choices (Frederick, Loewenstein & O'Donoghue, 2002). One example is choosing to spend a certain amount of money immediately or saving it for retirement. Another example is the choice between sticking to a healthy diet or eating a hamburger for lunch. In both examples, one option will fulfil one's needs instantly, while the other option will result in greater future satisfaction. In these kinds of choices, it has been observed that people are more likely to choose the situation where the reward is immediate but small, rather than to wait for the larger delayed reward. In the previous examples, this would mean that people are more likely to spend their money or to eat the hamburger immediately, instead of saving for retirement or continue with their diet for their future benefit.

To study these situations, intertemporal choice researchers use a task where participants have to choose between a smaller-sooner reward (SSR) and a larger-later reward (LLR). It has been proposed that people may be choosing the smaller-sooner reward over the larger-later one because the latter loses its reinforcing value as a result of its delay from the moment when the choice is being made (Ainslie, 1974; Mazur, 1984; Rachlin & Green, 1972). Specifically, the change in the value of a reward as a function of its temporal proximity is known as Delay discounting (Green, Fry & Myerson, 1994; Rachlin & Green 1972).

Value-based decision making is a process that involves decomposing the alternatives into a set of attributes. Based on these attributes, an alternative is evaluated and subsequently chosen or rejected (Bhatia, 2013; Bhatia & Stewart, 2018; Birnbaum & LaCroix, 2008; Rangel, Camerer, & Montague, 2008). This process can be described by decision models containing at least two elements[1]. The first one is an *integration rule* that assigns a value to the attributes of the available alternatives, considered to be the core element of the model. For example, the most commonly used models in intertemporal choice assume an integration rule that starts computing all the attributes of the same alternative into a single value to estimate the utility of alternatives. The second element is a *decision rule* that determines which alternative will be chosen, based on the values assigned by the integration rule. The model described on our previos example, would predict the election of the alternative with the highest value.

The objective of the present work was to model and to evaluate the integration rules used by the following two model families: the *alternative-based models* and the *attribute-based models*. The main discrepancy between the integration rules mentioned, is that the first one assumes that people assess options based on a single value assigned to each alternative, while the latter assumes people compare the individual attributes among each of the alternatives (Scholten, Read & Sanborn, 2014).

To illustrate the difference between both families, consider of a choice between a smaller-sooner reward (A= \$160 in 1 month) and a larger-later reward (B= \$300 in 4 months).

[1]See Dai & Busemeyer (2014), to explore other elements of decision models.

On the one hand, according to alternative-based models, the choice is made by assigning a subjective value to each alternative A and B, independently, each of which is discounted as a function of its delay. In other words, the amount of money offered is weighted by its delay to delivery, and once these subjective values are calculated, the alternative with the highest value is selected (Green & Myerson, 2004; Rachlin, Raineri & Cross, 1991). For example, assuming standard discount, alternative A would have a value of 139 dollars and alternative B would be worth 187 dollars; therefore, alternative B should be chosen.

On the other hand, according to attribute-based models, alternatives are directly compared among their attributes, and the one favoured by these comparisons is chosen. In other words, the difference between winning now or winning later will be compared against the difference between winning more or less (Scholten et al., 2014). Attribute-based functions weight the time advantage versus the magnitude advantage presented by each alternative. Continuing with the previously described pair of alternatives A and B, the amount difference would be 140 dollars and the time difference would be 3 months. In this case, it would be assumed that it is more advantageous to wait less time, therefore favouring the election of alternative A.

Another crucial difference between families is whether or not they assume *additivity in intervals*: alternative-based models do, while attribute-based models do not. According to alternative-based models, the total discounting observed over a certain interval should not depend nor be affected on whether and how this interval is subdivided. For example, the discount of 1 year should not depend on whether the year is divided into 12 months or not (Scholten & Read, 2010). Attribute-based models, on the other hand, do not imply the existence of additivity in intervals and therefore are able to describe two empirical phenomena that can not be explained by alternative-based models. These phenomena are known as *interval effects*, and they come in the form of subadditivity and superaddivity, depending on whether intervals are discounted more or less when they are segmented into smaller subintervals (McAlvanah, 2010; Scholten et al., 2014; Scholten & Read, 2006).

In the following sections, we will present a description of some of the most representative models of each family, followed by a detailed presentation of the additivity assumption.

Alternative-based models

In this family of models, the integration rule dictates assigning a subjective value to each alternative. Then, a possible decision rule is to choose the alternative with the highest value. Changes in the subjective value over time is assumed to be different on each model, and the shape of the function depicting this change is critical to describe several aspects of intertemporal choice (Green, Myerson & McFadden, 1997). The most representative models within the alternative-based family are called after the function shape they imply. These models are the Exponential (Samuelson, 1937), Hyperbolic (Mazur, 1984) and Hyperboloid (Green, Fry & Myerson, 1994), all of which have dominated the intertemporal choice literature (Urminsky & Zauberman, 2014).

Consider two alternatives, a smaller (x_s) sooner (t_s) reward, and a larger (x_l) later (t_l) reward. According to the Standard Economic Theory, the discounting utility of an alternative is given by the exponential function (Samuelson, 1937) as follows:

$$V_{SS} = (x_s \cdot e^{-k \cdot t_s}) \quad \& \quad V_{LL} = (x_l \cdot e^{-k \cdot t_l}) \tag{1}$$

In this equation, V_{LL} is the subjective value of the larger later reward and V_{SS} is the subjective value of the smaller sooner reward. The k parameter determines the rate at which value decreases with delay: a larger k is associated with steeper discounting, and a smaller k is associated with shallower discounting of the future reward's value. Once the subjective values of each alternative are computed, one possible decision rule would be to choose the one with greater value. However, assume the same scenario is presented repeatedly. Given this, the decision rule proposed by Luce (1959), which describes the choice as a probabilistic rather than an algebraic phenomenon, seems more appropriate:

$$P(LL) = \frac{V_{LL}}{V_{LL} + V_{SS}} \tag{2}$$

Exponential discounting predicts that individuals will have consistent preferences over time, assuming a constant discount rate (Green & Myerson, 1996; Green, Myerson & MacFadden, 1997). However, it has been observed that people are not consistent in their temporal preferences, leading to the Hyperbolic model being proposed as an alternative description of temporal discounting.

Considering the same elements from Equation 1, the Hyperbolic model is the following (Mazur, 1984):

$$V_{LL} = \left(\frac{x_l}{1 + k \cdot t_l}\right) \quad \& \quad V_{SS} = \left(\frac{x_s}{1 + k \cdot t_s}\right) \tag{3}$$

with the probability of choosing the larger-later reward expressed by Equation 2. The Hyperbolic discounting model implies that the discount rate diminishes with time, predicting a steeper rate of temporal discounting at the beginning, but a shallower discount rate for later delays; while the discount rate of the Exponential model remains constant thus time-independent (Green & Myerson, 1996). The difference in rates allows the Hyperbolic model to describe *preference reversals*, that is when subjects prefer a larger delayed good over a smaller, less delayed one, but as the passage of time makes the smaller one imminent, they change their preferences and select the smaller less delayed good (Killeen, 2009; Green, Fristoe & Myerson, 1994).

Another model that has been used to study temporal discounting is derived from assuming a hyperboloid function (Green, Fry & Myerson, 1994):

$$V_{LL} = \left(\frac{x_l}{(1 + k \cdot t_l)^{\tau}}\right) \quad \& \quad V_{SS} = \left(\frac{x_s}{(1 + k \cdot t_s)^{\tau}}\right) \tag{4}$$

This function includes an additional free parameter τ that represents the nonlinear scaling of amount and time, for which a value smaller than 1 allows the discounting curve to decrease subtly with larger delays. Notice that when τ equals 1.0, Equation 4 is simplified to Equation 3. Numerous studies have been conducted to compare the adjustment of these three functions to different sets of discounting data, and almost invariably, the Hyperboloid model seems to provide a better fit, often even after controlling for its additional free parameter (Green, Myerson & Vanderveldt, 2014; McKerchar et al., 2009; Myerson & Green, 1995).

Attribute-based models

In this family of models, the integration rule computes the difference between the pair of attributes, delays and outcomes. This will lead to each alternative to be 'better" in terms of one of these attributes, and the decision rule will favour the alternative that presents the greater gain on either time or outcome. There are several models in this family, the present study describes three: the Trade-off model (Scholten et al., 2014), the ITCH model (Ericson et al., 2015) and the Direct Differences model (Dai & Busemeyer, 2014; González-Vallejo, 2002).

The Trade-off model (Scholten et al., 2014) describes intertemporal choice to be governed by an attribute-based integration rule, which weights the difference in outcome against the difference in delay, choosing the alternative with the greater difference on its favor. According to this model, the probability of choosing the larger-later reward is described by the following equation:

$$P(LL) = \frac{((v(x_l) - v(x_s))^{\frac{1}{\varepsilon}}}{(v(x_l) - v(x_s))^{\frac{1}{\varepsilon}} + (Q(w(t_l) - w(t_s)))^{\frac{1}{\varepsilon}}} \tag{5}$$

where ε is a noise parameter. $Q(w(t_s), w(t_l))$ represents the trade-off function, which has an S-shape over intervals, allowing to account from superadditive (ϑ) to subadditive (α) discounting rates as intervals' length increase:

$$Q(w(t_s), w(t_l)) = \frac{\kappa}{\alpha} log\left(1 + \alpha\left(\frac{w(t_l) - w(t_s)}{\vartheta}\right)^{\vartheta}\right) \tag{6}$$

where κ is a discount parameter. Changes on the subjective value of the outcomes and time attributes is captured within Equation 6 by functions $v(x)$ and $w(t)$, respectively. These functions are concave when τ or γ have high values and linear when they are close to zero, as in the following equations:

$$v(x) = \frac{1}{\gamma} log(1 + \gamma x) \quad \& \quad w(t) = \frac{1}{\tau} log(1 + \tau t) \tag{7}$$

Another model that uses the differences between attributes to choose between alternatives, is the ITCH model (Intertemporal Choice Heuristic), defined by Ericson, White, Laibson and Cohen (2015) as:

$$P(LL) = \tag{8}$$

$$L\left(\beta_0 + \beta_{x_A}(x_l - x_s) + \beta_{x_R}\frac{x_l - x_s}{x*} + \beta_{t_A}(t_l - t_s) + \beta_{t_R}\frac{t_l - t_s}{t*}\right)$$

where β_0 is the intercept, $_R$ stands for Relative and $_A$ for Absolute; $x*$ and $t*$ represent a reference point that is the arithmetic mean of the two alternatives along each dimension: $x* = \frac{x_s + x_l}{2}$, $t* = \frac{t_s + t_l}{2}$. In this model, L is the cumulative distribution function of a logistic distribution with mean 0 and variance of 1. Thus, each term used in the model represents either an absolute or a proportional arithmetic operation that compares the two alternatives along a particular dimension or attribute (outcome or time). Each term is multiplied by parameter β, that represents the weight given to either heuristic rule when deciding between the two alternatives. The weighted sum of the outcomes predicted by each heuristic determines the probability of choosing the larger-later reward.

The Direct Differences (DD) model is a combination of two different equations: the integration rule from Dai and Busemeyer (2014) and the decision rule proposed by González-Vallejo (2002):

$$P(LL) = \Phi\left(\frac{d - \delta}{\sigma}\right)$$

where

$$d = w(x_l - x_s) - (1 - w)(t_l - t_s) \tag{9}$$

with d representing the difference among alternatives, and w as the amount of attention allocated to the magnitude attribute, and $1 - w$ to the delay attribute. The original equation -proposed by Cheng & González-Vallejo (2016) to describe intertemporal choices- considers the same decision rule but its integration rule, which contains two-dimensional choice options, is defined as follows:

$$d = \left(\frac{max\{|x_l|, |x_s|\} - min\{|x_l|, |x_s|\}}{max\{|x_l|, |x_s|\}}\right) - \left(\frac{max\{|t_l|, |t_s|\} - min\{|t_l|, |t_s|\}}{max\{|t_l|, |t_s|\}}\right) \tag{10}$$

The Direct Differences model differs from the Proportional Differences (PD) model in the integration rule each one assumes. The first model takes into consideration the direct differences among attributes, while the second one considers relative differences. The decision rule, the probability of choosing the larger-later reward, represents the overall advantage of the LL option over the SS option, and Φ corresponds to the cumulative distribution function of a standard normal distribution. The decision threshold, δ, is a free parameter that captures the relative importance that a given decision-making agent assigns to the attribute differences, as well as the differential weight given to each attribute involved in the decision process. It is usually stated that $\delta < 0$ indicates a preference for selecting one attribute (either delay or outcome), and when $\delta > 0$ this preference seems to change for selecting the other attribute. Finally, the σ parameter is a measure of the variability presented in the utility difference.

Additivity

The most important derived assumption that distinguishes alternative-based models from attribute-based models, is the existence of additivity: the first models assume additivity, while the latter ones do not. The assumption of additivity in intervals entitles that the total discounting over an interval should not depend on whether and how this particular interval is subdivided (Cheng & González Vallejo, 2016; Read, 2001; Scholten, & Read, 2010).

To illustrate additivity in intervals we present Figure 1 where each of the 4 bars shown represents a question, the leftmost end corresponds to the smaller-sooner reward, and the rightmost end the larger-later reward. Delays and outcomes are indicated in the superior axis and the letters A, B, C and D are used to identify each alternative. Question 1, for example, offers A=*$ 5150 pesos in one week* against B=*$ 5300 pesos in two weeks*. This graphic represents two different procedures: a segmented one and non-segmented one. Questions 1, 2 and 3 are the segmented procedure where alternatives are divided into subintervals, while question 4 is the non-segmented procedure by presenting an undivided alternative that covers all of the subintervals. According to the additivity assumption, the two procedures must have the same discount overall, in other words, the discount is additive when both procedures result in the same subjective value (Scholten & Read, 2010). Attribute-based models do not assume additivity by proposing that subjective values are not just computed as a function of delays but of the distance between the delays presented for each alternative (Read, 2001).

One measure used to account for additivity is the Discount fraction (Cheng & González-Vallejo, 2016) of an interval:

$$F[t_A, t_B] = \frac{1 + \kappa * t_A}{1 + \kappa * t_B} \tag{11}$$

which represents the proportion of the money (magnitude of

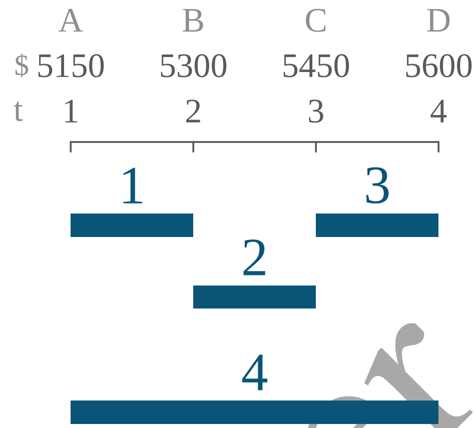

Figure 1. Graphical representation of an intertemporal choice situation. Each bar represents a question, the leftmost end indicates the smaller-sooner reward and the rightmost end the larger-later reward. Questions 1, 2 and 3 represent the segmented procedure; question 4 represents the un-segmented procedure.

the reward) that is subjectively left after this interval. Following with our example, equation 11 is the discount fraction of the inverval covered from alternatives A and B (question 1). Altogether, in order to be consistenty with the assumption of additivity in intervals, the following equation must be satisfied:

$$F[t_A, t_D] = F[t_A, t_B] * F[t_B, t_C] * F[t_C, t_D] \tag{12}$$

Where the discount fraction of a complete interval (question 4, referred to as $F[t_A, t_D]$), is assumed to be equivalent to the product of the discount fractions of the individual subintervals (questions 1, 2, and 3). If this condition is satisfied, then the decision-making agent should choose the same alternative when they are presented with the intervals and the corresponding subintervals. This would mean that, for example, if a person has chosen the larger-later reward in the subintervals, they must choose the LL alternative again when presented with the complete interval, and the same way with SS alternatives. In other words, choices observed when subintervals are presented, are expected to be consistent with the choice made about the complete interval.

However, some studies have reported choices that are inconsistent with this assumption by finding evidence of Interval effects (McAlvanah, 2010; Scholten & Read, 2006; 2010; Read, 2001), defined as:

Superadditivity: A greater discount over complete intervals than the one observed for their corresponding subintervals. Empirically, this would mean choosing the smaller-sooner reward on the full intervals (for example, question 4), but the larger-later one on its corresponding subintervals (questions 1, 2 and 3).

Subadditivity: A great discount over subintervals than the one observed for the corresponding full intervals. Empirically, this would mean choosing the smaller-sooner reward in subintervals (for example, in questions 1, 2 and 3), but the larger-later one on the corresponding full intervals (question 4).

Choice Variability

Studies that have tested attribute-based models do not always consider the stochastical nature of choice, that has been observed within individual preferences when the same pair of alternatives is presented several times. To have repetitions of alternatives as part of the experimental protocol allows separating the behavioral variability from the structural inconsistency of preferences. As a matter of fact, in studies where choice variability is taken into consideration, subjects tend to present preferences that are consistent with the additivity assumption (Cavagnaro & Davis-Stober, 2014; Dai, 2016; González-Vallejo, 2002; Myung, Karabatsos & Iverson, 2005; Regenwetter, Dana & Davis-Stober, 2011; Regenwetter & Davis-Stober, 2012). It is important to note that in this type of studies, the inconsistency of preferences was assessed concerning the transitivity axiom (for alternatives A, B and C, a preference for A > B and B > C implies the preference for A > C) and its stochastical derivatives. The violation of the transitivity axiom known as intransitivity, (Tversky, 1969) could be understood as similar to the violation of the additivity assumption, but in terms of risky rewards (Scholten et al., 2014).

Additionally, studies that have reported evidence of interval effects, have analyzed their data by using a single parameter to describe the behavior of all subjects. However, it has been noted before that results derived from a pooled analysis can be substantially different from those computed at the individual level (Estes, 1956; Wagenmakers, Lee, Lodewycks & Iverson, 2008). This can be illustrated by the Condorcet paradox, which refers to a scenario where individual preferences conflict with majority preferences, especially when the preferences differ from one individual to another. Actually, it has been reported that when data is analyzed at the individual level, fewer instances of inconsistency are found (Dai, 2016; Dai & Busemeyer, 2014; Regenwetter, Dana & Davis-Stober, 2010). Altogether, these findings stress the great importance of looking at the individual data and its relation to population-level behavior. This allows us to draw conclusions about the empirical phenomena and the processes behind them, and not about the way in which data are analyzed.

Alternative-based models have dominated the intertemporal choice literature. However, only the attribute-based models have been able to account for interval effects. Considering both families of models and the different approximations given to the preference inconsistency problem (such as intransitivity), interval effects have not been able to be replicated when choice variability and individual data are taken into account. Given the variability of results reported around this topic, we decided to do an experiment to elicit interval effects, considering choice variability and data analysis at the individual level. Additionally, we studied and compared the descriptive adequacy of different models, with the application of Bayesian methods.

Method

Participants. 25 undergraduate students from the School of Psychology at the National Autonomous University of Mexico. For their participation, they entered a raffle where they could win a Netflix, iTunes or Spotify $300 MXN gift card, depending on their preference.

Procedure. The procedure was conducted in one session that lasted about 35 minutes. Each participant performed the experimental task in a desk computer located in a closed room without noise. Prior to the experiment, all subjects read and signed an informed consent form. The task was developed in PsychoPy v1.83.04 (Pierce, 2007), with the locations of the smaller-sooner and the larger-later alternatives at the left or right side of the screen, randomized across trials (see Appendix A for instructions).

Experimental Design. The task was based on the second study presented in Scholten et al. (2014). The experimental design consisted of 12 fixed alternatives that have linear increments within them: $150 Mexican pesos for outcomes and one week for time. Large outcomes ranged from 5150 to 6500, and small ones from 1150 to 1450, while intervals ranged from one to 10 weeks. A total of 22 different questions were created by combining different pairs of alternatives, classified in four sets: 1) Small intervals, 2) Medium intervals, 3) Long intervals/Large outcomes, and 4) Long intervals/Small outcomes (see a graphical representation of the experimental design in Appendix B, Figure 7). To consider choice variability, each question was presented 10 times, leading to a total of 220 trials. The method used allows the assessment of responses obtained at both, intervals and subintervals so, for example, questions 1, 2, 3, 7, 8 and 9 consider the shortest intervals and they can all be understood as subintervals of question 18, which considers the longest interval.

Bayesian Cognitive Modeling

The evaluated models were 1) Hyperboloid, 2) ITCH, 3) Trade-off, 4) Direct Differences and 5) Proportional Differences, the latter being considered a derivate of the fourth model.

Two of the greatest advantages of the application of Bayesian methods are that they capture the uncertainty behind each parameter values in posterior densities while enabling the estimation of parameters at both individual and group levels, without changing the assumptions of the mathematical model (Lee, 2018; Nilsson, Rieskamp & Wagenmakers, 2011; Wagenmakers et al, 2016; 2008). There are examples in the literature of the application of Bayesian methods to intertemporal choice, which illustrate the advantages described before (see Chávez et al, 2017; Vincent, 2016).

The used notation for describing Bayesian analysis was adopted from Lee & Wagenmakers (2014). In this type of representations, often known as Graphical models, shaded nodes indicate observed variables, and unshaded nodes stand for latent variables. Double-bordered nodes indicate a deterministically computed parameter, while single-bordered nodes imply stochasticity. Circle-like nodes represent continuous variables, and squares-like nodes portray discrete variables.

All models share the following core structure: They have three rectangles (known as "plates") that indicate independent replications of the intertemporal choice process assumed by each model, for every 1) participant, i; 2) question, j; and 3) the repetitions of the same question, r. Individual responses, C_{ijr}, were modelled as a Bernoulli process with parameter θ_{ij}, that represents the probability of choosing the larger-later reward. Node x^s_{ij} represents the smaller amount of money presented on each independent trial, while x^l_{ij} correspond to the larger amount of money presented. On the other hand, node t^s_{ij} indicates the sooner delay presented and t^l_{ij} the later delay.

Hyperboloid Model. Figure 2 is the Bayesian graphical model of the hyperboloid function (Equation 3), where the probability of choosing the larger-later reward, θ_{ij}, depends on the decision rule proposed by Luce (1959); however, this version adds a noise parameter, ϵ, to allow for stochastic error (Andersen et al, 2010); in the present work this parameter was not estimated individually. The discounted values, v^{ll}_{ij} and v^{ss}_{ij}, are obtained by multiplying the discount factor by the outcome. The discount factor, d^{sl}_{ij}, has the hyperboloid structure with two parameters: κ_i and τ_i, estimated individually and with a normal distribution prior with mean 0 and a standard deviation of 1, truncated for positive values. It is important to mention that the discount factor of this model could be easily replaced with other types of functions used in the alternative-based family, such as the exponential and the hyperbolic functions, without modifying any other aspect of the model. These last two models, as well as several versions of them were evaluated, but since their performance was low and fairly similar to one another, we decided to consider only the hyperboloid function results.

Trade-off Model. The Trade-off model (Figure 3 and Equations 5, 6 and 7) uses the same decision rule as the

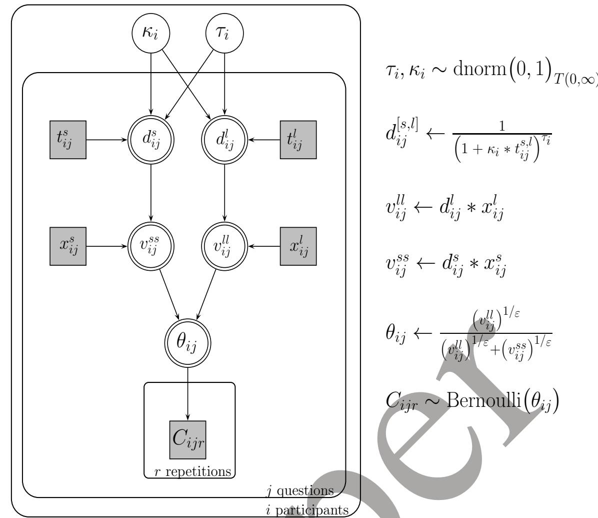

Figure 2. Bayesian Cognitive Modeling of the Hiperboloid Model

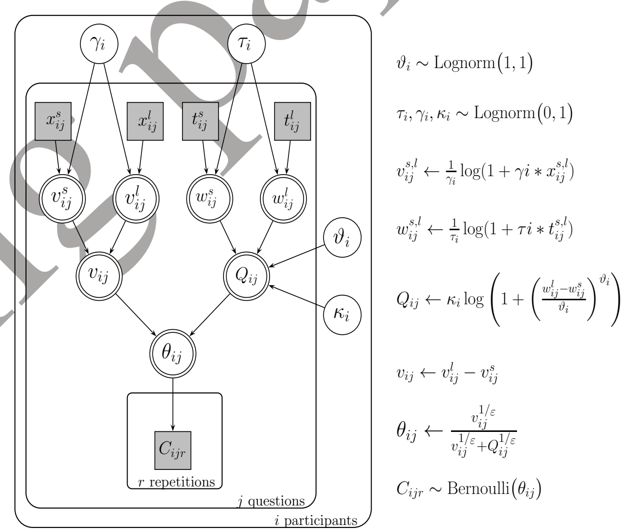

Figure 3. Bayesian Cognitive Modeling of the Trade-off Model

Hyperboloid model θ_{ij} probability. In this model, Q_{ij} represents the Trade-off function which contains parameters κ_i and ϑ_i that capture the differences between the weighted delays. The delays' weights are assigned by $w^{[s,l]}_{ij}$ which considers the influence of parameter τ that diminishes the weight of time. The same mathematical structure is applied to decrease the subjective value of the outcomes but with parameter γ_i. The original Trade-off (Equation 6) function contains parameter α which accounts for subadditivity, was not considered in the analysis because our data did not show evidence of subadditivity and its inclusion seemed to increase variability in the parameter estimation of the model. All free parameters were estimated at the individual level, except for ϵ, by using a lognormal prior distribution with mean 0 and standard deviation of 1 (except for ϑ which used a mean of 1). This model was based on the statistical work presented by Scholten et al., (2014) where they conducted a model comparison with Bayesian statistical tools, assuming all participants' performance could be described with a single group-level parameter. In contrast, in the present study we assumed that each participant's response pattern could be described by its own parameter distribution.

ITCH. In this model (Figure 4 and Equation 8), the probability of choosing the larger-later reward, θ_{ij}, is defined by Φ, the cumulative distribution function of a standard normal distribution [2]. The probit function takes into consideration an intercept term (β_0), and two types of differences: absolute d^A and relative d^R. These differences are assigned to outcome and delay, separately; with parameters β_i^{xA}, β_i^{xR}, β_i^{tA} and β_i^{tR} representing the weight given to each difference. All parameters have a normal prior distribution, with mean 0 and a standard deviation of 1.

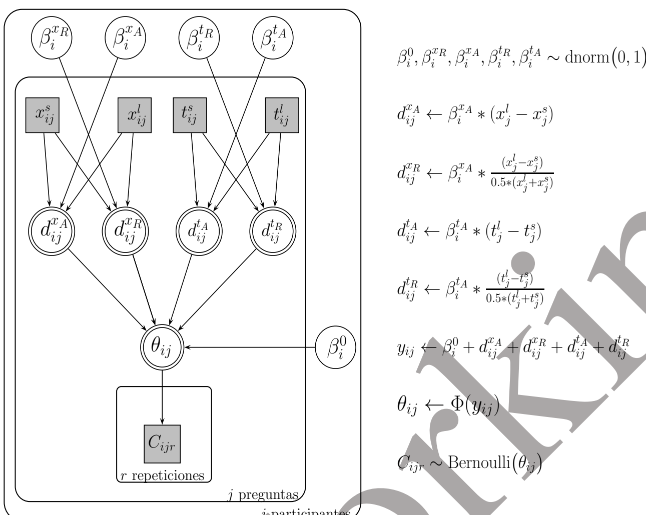

Figure 4. Bayesian Cognitive Modeling of the ITCH model

Direct Differences. Figure 5 presents the Direct Difference model described by Equation 9. In this case, the probability of choosing the larger-later reward is also defined by the cumulative normal distribution. Parameter δ_i captures the decision threshold that indexes the relative importance assigned to time and outcome attributes to produce of a final choice; parameter σ_i is the standard deviation which indicates internal variability. The model weights the differences through w_i, the amount of attention being allocated to each attribute, where w_i is pondered by outcome differences and $1 - w_i$, by delay differences. For the prior distributions, we used a normal distribution with mean 0 and a standard deviation of 1; however, σ is truncated to positive values.

Proportional Differences. This model was implemented and evaluated by using the same structure as the Direct Differences model but, in this case, the assignation of the d_{ij} node followed relative differences (Equation 10) instead of direct differences, so parameter w was not estimated[3].

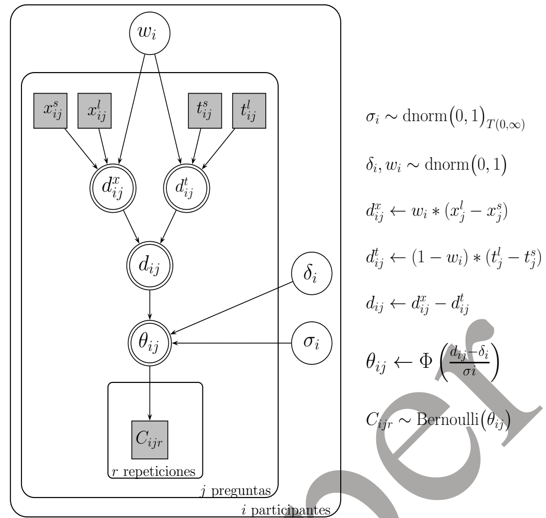

Figure 5. Bayesian Cognitive Modeling of the Direct Differences model

Results

The top-left panel of Figure 6 displays the observed data for all 10 repetitions presented per question, with blue bars representing a choice of the LLR and red bars representing the SSR choice. Each row of data corresponds to a different participant, who were ordered according to their proportion of larger-later choices. Questions are arranged in terms of the interval length, grouped in the four sets described previously, which are signalled at the top of the graphic. For example, participants 6, 2 and 7, located at the bottom part of the graphic, always chose the SSR in small, medium and large intervals. Similarly, participants 4, 17 and 9, located at the upper part of the graphic, chose the LLR regardless of the interval length. There are some participants that chose the LLR in the three sets presenting large outcomes but chose otherwise for the small outcomes set, such as participants 5 and 8. These patterns follow the additivity assumption since participants chose the same type of alternative (LLR or SSR)

[2]The original model uses the cumulative distribution of a logistic distribution with a mean of 0 and variance of 1. However, in the present work, we used the probit model. Several authors have compared logit vs probit models and have concluded that both lead to similar fits (Agresti, 2007; Chambers & Cox, 1967; Hahn & Soyer, 2005).

[3]Information regarding models, analysis and results can be found in the following repo: https://github.com/ElenaVillalobos/InterTempoChoiceModels

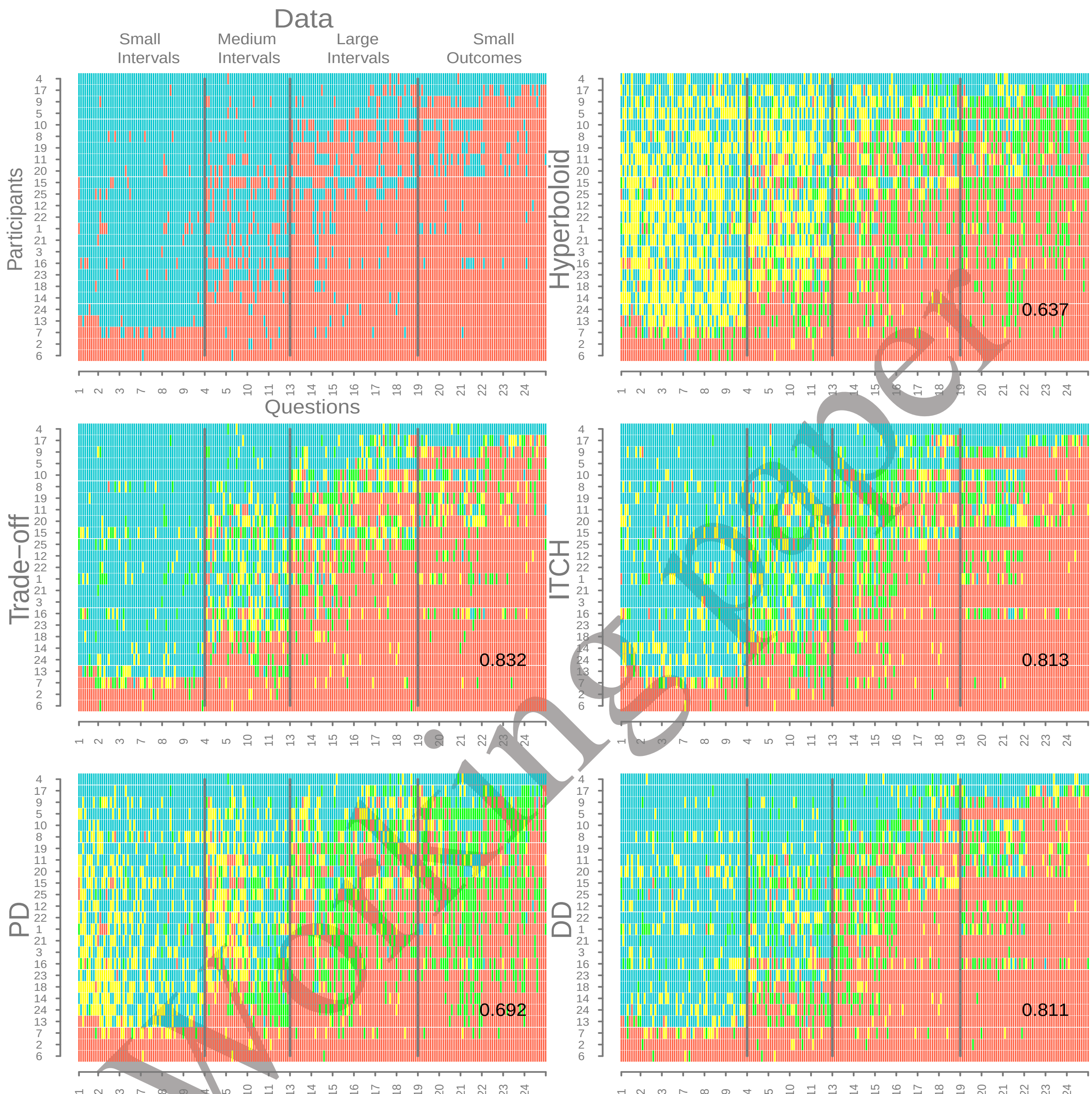

Figure 6. Choice data and model predictions. Each panel presents the choices registered for each of the 10 repetitions of the 22 questions of the experimental task. The X-axis shows questions according to the length of the interval, with a grey line signaling the four segments previously described. The Y-axis shows participants arranged by the total number of choices made for the LLR. In the top-left panel, red is the choice for the SSR and blue for the LLR. The remaining panels contrast the predictions made by the evaluated models, identified along the Y-axis; green is the model prediction of an LLR choice when the actual choice was the SSR; likewise, yellow is the prediction of an SSR choice when the selection was the LLR; red is the correct prediction of an SSR choice and blue of an LLR one. The proportion of good prediction is shown at the bottom right of each panel.

regardless of interval size. It is important to note that when the set of small outcomes was presented, almost all participants chose the SSR with an unclear interval size effect.

Participants displayed at the central rows, show at least one response pattern that is consistent with attribute-based models, by showing a dependency on interval size. For example, Participant 24 chose the LLR in the small intervals but the SSR with the medium and large intervals, a response pattern showing superadditivity. Similarly, other participants presented patterns where, as the length of the intervals increased, their choices changed from choosing the LLR to the SSR. These individual differences stress the importance of finding a model that can describe patterns that are consistent with both presence and absence of interval effects.

Figure 6 also shows predictions from each model and its correspondence to the observed data, with yellow and green bars indicating mispredictions made by the models, when the observed data corresponded to the election of the larger-later reward and the smaller-sooner reward, respectively. The top-right panel shows the predictions made by the Hyperboloid model, with a concentration of yellow bars displayed at the small and medium interval sets, indicating that the model predicted the choice of SSR when LLR was selected. On the other hand, in the small outcomes set there was a greater presence of green bars, indicating that the model predicted choices of the LLR when in fact the SSR was chosen. From all tested models, this was the model with the lowest percentage of correct predictions (63%). The high rate of mispredictions made by this model is thought to be related to the fact that it assumes additivity in intervals.

The Proportional Differences model (bottom-left panel) had the second-worst prediction rate (69%). A misprediction pattern can be observed for the small and medium intervals, where there is a predominant presence of yellow and blue bars, whereas for the large intervals and small outcomes, more red and green bars are displayed. Altogether, this indicates that despite the PD model being an attribute-based model, it performs worse than the other tested models from this family.

The ITCH and the Direct Differences models performed similarly, both had 81% of correct predictions. Besides, both models mispredicted smaller-sooner participants' choices for the medium and large intervals (more presence of green bars in these sets) but showed good prediction for the small intervals and small outcomes sets (low presence of yellow bars). Although the difference could be considered marginal, it seems like the DD model has a bit more of misprediction than the ITCH model, with none of them presenting a specific and clear pattern in their mispredicted choices.

The Trade-off model showed the highest correct-prediction percentage (83%) with a very low presence of misprediction for small intervals, but not so in the medium and large intervals, where more misprediction (presence of green and yellow bars) is found. In the small outcomes set, there seemed to be more mispredictions, even if all participants chose the smaller-sooner reward across trials.

The Trade-off, ITCH and DD models performed similarly; therefore, the following section analyses the parameters of each model to contrast their descriptive adequacy to the observed data.

Parameter evaluation

To find the model that better described our data, a parameter analysis was conducted, and whose results are displayed in Figures 8, 9 and 10 (in Appendix B). Figure 8 presents the posterior distributions computed for the parameters of the Trade-off model for each participant, with lines representing the parameter values covered by the posterior and a single point used to indicate the location of the posterior's mean; with participants ordered according to ϑ values. In this figure, we can observe parameter τ has a narrow range for all posterior distributions, except for participants 2, 6, and 9, which have a wider range. Meanwhile, parameter ϑ describing superadditivity seems well defined for all participants. Parameters γ and κ show values higher than 80 for some participants, especially for participants with higher values of ϑ (located at the bottom of these plots). This could indicate that the model might be over-parametrized since posterior distributions covering higher and wider ranges of values could be a sign of multiple different ways in which model parameters can be recovered from the generated data-set. Additionally, there are no clear groups formed within the parameters estimated for all participants. The simulations obtained were not able to recreate data patterns similar to the ones observed in the real data, by using parameter values that correspond with the mean of the posterior distributions estimated from our data.

Figure 9 shows the corresponding posterior distributions for the ITCH model for each participant. In this case, participants were ordered according to parameter β_x^A estimations. There are no clear clusters detected except for the suggested correlation between β_t^A and β_x^A: when participants have a narrow posterior with values close to zero for β_x^A, they tend to have a narrow and close to zero posterior distribution for β_t^A, and vice-versa; participants have a wide posterior in both parameters. All posterior distributions computed for β_t^R are narrow and have negative values. However, posterior distributions for β_x^R can be considered troublesome, since the means of all participants are very similar and cover the range of values established by the prior distribution. This could be an indicator of the relative outcome differences not giving enough information about this parameter. This result was revised in greater detail by running further simulations that demonstrated that only big changes in the relative β's value could generate different data patterns. Altogether, these findings suggest this model might be over-parametrized and that relative differences are not very informative.

Figure 10 presents the posterior distributions computed for the Direct Differences model, with participants arranged according to δ posterior values. According to these plots, there seems to be a relation between parameters σ and δ. When δ values are less than -2.5, a greater variability in σ can be observed, with values usually larger than 1. When δ is greater than -2.5, σ posterior distributions seem to narrow below 1. In regards to parameter w, all posterior distributions were narrow, except for participants 6 and 2. It is interesting to note that clusters obtained by simulations are fairly close to those found in the real data.

Put together, these results showed that the model that better described the present data is the Direct Differences one. This model seemed to allow for a more appropriate parameter interpretation for the present data set, especially when compared to the ITCH and Trade-off models whose interpretation was difficult to generate. Besides, according to the model simulations computed, the Direct Differences model proved to replicate our data, while the ITCH and Trade-off models failed to do so.

Discussion

The main goal of the present work was to study and compare five different models from the intertemporal choice literature: the Hyperboloid, the Trade-off, the ITCH, the Proportional Differences and the Direct Differences; the last four can to account for interval effects.

In the data from the present study, some participants showed evidence of changing their choices depending on the length of the interval involved in the questions presented, while others did not. Most participants that displayed an interval effect, presented the superadditivity pattern, by choosing the larger-later reward in the small interval set, but the smaller-sooner reward in the large interval set. It is worth noting that a reduced number of participants proved to be consistent with the assumption of additivity in intervals, by choosing either the larger-later or the smaller-sooner reward regardless of interval length.

We also found greater choice variability for the medium size set than for the small or large interval sets. These results seem to concur with research that has found it is harder for participants to decide between alternatives with similar or closer attribute values (REFERENCIA). The inclusion of 10 repetitions of each question designed for the present study, allowed us to examine choice variability and structural inconsistency of preferences separately.

The Hyperboloid model, even though it had the best performance within alternative-based family, it performed the worst in comparison to the attribute-based models tested in the present study; other publications have reported similar results (Chen & González-Vallejo, 2016; Ericson et al., 2015; Dai & Busemeyer, 2014; Scholten et al., 2014). According to the Hyperboloid model, the probability of choosing either the LLR or SSR was of 0.5, even when actual choices were favouring a certain type of alternative in all repetitions. The most likely explanation for this is the additivity assumption contained in the model.

The Trade-off model was developed to account for interval effects; however, it is not free of limitations. First, it has a complex mathematical structure and contains several parameters that seem to be explaining the same part of the decision process, also known as over-parametrization, which goes against the parsimony principle that holds that models should be simple and avoid unnecessary assumptions. Second, it proved not to be sensitive to the observed differences in the choices between large and small outcomes sets, when the interval lengths were the same. According to this model, if a participant selected the larger-later reward in the medium and large interval sets, this should remain unchanged for intervals of the same length from the small outcomes set. This means, that the model makes the same predictions for intervals of the same length regardless of the magnitude of the alternatives presented. However, as observed in the present data set, participants always chose the smaller-sooner reward in the small outcomes set, regardless of interval length.

The ITCH model includes an integration rule also based on attributes, but which takes into consideration the relative and absolute difference computed between them. In the present study, parameters related to the relative differences showed difficulties in terms of the computation of informative posterior distributions from the data obtained. This could indicate that the model is over-parametrized, so that proportional differences are not providing additional information to what was already obtained from absolute differences. This was confirmed by looking at the Proportional Differences model performance, which presented the lowest percentage of correct predictions from the attribute-based models, while only considering relative differences.

The Direct Differences model does not have a complex mathematical structure and is the simplest one from the attribute-based models, tested in the present work. There are three main finding from this model. First, it includes an integration rule based on attributes and its percentage of correct prediction was equally high as the ITCH model, without having as many parameters. Second, it assumes that perceived differences between attributes are captured by a stochastic process, with the choice probability increasing monotonically as a function of the direct differences. For example, if attention is drawn to delay, participants are expected to choose the immediate reward; however, if their attention is allocated to outcome, then people would be expected to select the larger reward, despite its delay. Third, parameter estimation allowed us to observe that some participants showed a correlation between parameters, while others did not; a relation that concurs among participants that behave similarly.

Conclusions

In conclusion, the attribute-based rules seem more adequate to describe the presence and absence of interval effects. In the present study, the model that worked better with the present database was the Direct Differences model. However, it is important to further study this model to identify which intertemporal choice properties have a systematic effect on its parameters.

The more complex the model, the more difficult it is to withdraw relevant and meaningful interpretations, even if the percentage of correct predictions is high. This casts some doubt on the extent to which models are able to describe and make sense of the data obtained.

The models included in the present work are different, eventhough some use the same integration rule. It was difficult to compare their performance usign a singular measure. However, the Bayesian revision of the parameters computed for each model allowed us to elaborate more significant conclusions about the theoretical implications and performance of each one, providing us with the opportunity to identify that better described the present data set.

Even if alternative-based models proved to be incapable of accounting for the present data set, these models have been very important in the literature because they correlate with risky behaviors, such as tobacco or alcohol comsumption. It only seems natural that the next step, for attribute-based models, is to demonstrate whether or not they can be useful to describe relevant everyday behaviors.

References

Agresti, A. (2018). An introduction to categorical data analysis. John Wiley & Sons.

Ainslie, G. W. (1974). Impulse control in pigeons. *Journal of the experimental analysis of behavior,* 21(3), 485-489.

Andersen, S., Harrison, G. W., Lau, M. I., & Rutstrom, E. E. (2010). Behavioral econometrics for psychologists. *Journal of Economic Psychology*, 31(4), 553-576.

Bhatia, S. (2013). Associations and the accumulation of preference. *Psychological Review,* 120(3), 522–543.

Bhatia, S., & Stewart, N. (2018). Naturalistic multiattribute choice. *Cognition*, 179, 71-88.

Birnbaum, M. H., & LaCroix, A. R. (2008). Dimension integration: Testing models without trade-offs. *Organizational Behavior and Human Decision Processes*, 105(1), 122-133.

Cavagnaro, D. R., & Davis-Stober, P. C. (2014). Transitive in Our Preferences, but Transitive in Different Ways: An Analysis of Choice Variability *Decision,* 1(2), 102-122.

Chávez, M. E., Villalobos, E., Baroja, J. L., & Bouzas, A. (2017). Hierarchical Bayesian modeling of intertemporal choice. *Judgment and Decision Making,* 12(1), 19.

Cheng, J. & González-Vallejo, C. (2016). Attribute-Wise vs Alternative-Wise Mechanism in Intertemporal Choice: Testing the Proportional Difference, Trade-Off and Hyperbolic Models. *Decision*, 3(3), 190-215.

Chambers, E. A., & Cox, D. R. (1967). Discrimination between alternative binary response models. *Biometrika,* 54(3-4), 573-578.

Dai, J. (2016). Are Intertemporal Preferences Transitive? A Bayesian Analysis of Repeated Individual Intertemporal Choices *Decision,* 1-24.

Dai, J. & Busemeyer, J. R. (2014). A Probabilistic, Dynamic and Attribute-Wise Model of Intertemporal Choice *Journal of Experimental Psychology: General,* 143(4), 1489-1514.

Ericson, K. M., White, J. M., Laibson, D., & Cohen, J. D. (2015). Money earlier or later? Simple heuristics explain intertemporal choices better than delay discounting does. *Psychological Science,* 26(6), 826-833.

Estes, W.K. (1956). The problem of inference from curves based on group data. *Psychological Bulletin,* 53, 134-140.

Frederick, S., Lowenstein, G & O'Donoghue, T. (2002). Time Discounting and Time Preference: A Critical Review *Journal of Economic Literature*, 40(2), 351-401.

González-Vallejo, C. (2002). Making trade-offs: A probabilistic and context-sensitive model of choice behavior. *Psychological Review,* 109 (1), 137-155.

Green, L., Fristoe, N., & Myerson, J. (1994). Temporal discounting and preference reversals in choice between delayed outcomes. *Psychonomic Bulletin & Review,* 1(3), 383-389.

Green, L., Fry A. F., & Myerson, J. (1994). Discounting of Delay Rewards: A Life-Span Comparison. *Psychological science,* 5(1), 33-36.

Green, L., & Myerson, J. (2004). A Discounting Framework for Choice with Delayed and Probabilistic Rewards *Psychological Bulletin,* 130(5), 769-792.

Green, L., & Myerson, J. (1996). Exponential Versus Hyperbolic Discounting of Delayed Outcomes: Risk and Waiting Time. *American Zoologist,* 36(4), 496-505.

Green, L., Myerson, J., & McFadden, E. (1997). Rate of temporal discounting decreases with amount of reward. *Memory & Cognition,* 25(5), 715-723.

Green, L., Myerson, J., & Vanderveldt, A. (2014) Delay and Probability Discounting. In: McSweeney, F. K. & Murphy, E. S. (Eds.). *The Wiley Blackwell Handbook of Operant and Classical Conditioning.* John Wiley & Sons, Malden, MA.,307-337.

Hahn, E. D., & Soyer, R. (2005). Probit and logit models: Differences in the multivariate realm. The Journal of the Royal Statistical Society, Series B, 1-12.

Killeen, P. R. (2009). An additive-utility model of delay discounting. *Psychological Review,* 116(3), 602-619.

Lee, M. D. (2018). Bayesian methods in cognitive modeling. *Stevens' Handbook of Experimental Psychology and Cognitive Neuroscience,* 5, 1-48.

Lee, M.D. & Wagenmakers, E.-J. (2014). *Bayesian cognitive modeling: A practical course.* Cambridge: Cambridge University Press.

Luce, R. D. (1959). *Individual choice behavior.* New York: Wiley.

Mazur, J. E. (1984). Test of an Equivalence Rule for Fixed and Variable Reinforcer Delay. *Journal of Experimental Psychology: Animal Behaviour Processes,* 10(4), 426-436.

McAlvanah, P. (2010). Subadditivity, patience, and utility: The effects of dividing time intervals. *Economic Behaviour & Organisation*, 325-337.

McKerchar, T. L., Green, L., Myerson, J., Pickford, T. S., Hill, J. C., & Stout, S. C. (2009). A comparison of four models of delay discounting in humans. *Behavioural Processes*, 81(2), 256-259.

Myerson, J., & Green, L. (1995). Discounting of Delayed Rewards: Models of individual choice. *Journal of the Experimental Analysis of Behaviour*, 64(3), 263-276.

Myung, J. I., Karabatsos, G., & Iverson, G. J. (2005) A Bayesian approach to testing decision making axioms *Journal of Mathematical Psychology*, 49(3), 205-225.

Nilsson, H., Rieskamp, J., & Wagenmakers, E. J. (2011). Hierarchical Bayesian parameter estimation for cumulative prospect theory. *Journal of Mathematical Psychology*, 55(1), 84-93.

Pierce, J, W. (2007). PsychoPy-Psychophysics software in Python. *Journal of Neuroscience Methods,* 162(2), 8-13.

Rachlin, H., & Green, L. (1972). Commitment, Choice and Self-Control. *Journal of the Experimental Analysis of Behaviour,* 17(1), 15-22.

Rachlin, H., Raineri, A., & Cross, D. (1991). Subjective Probability and Delay. *Journal of the Experimental Analysis of Behaviour,* 55(2), 233-244.

Rangel, A., Camerer, C., & Montague, P. R. (2008). A framework for studying the neurobiology of value-based decision making. *Nature reviews neuroscience*, 9(7), 545-556.

Read, D. (2001). Is time-discounting hyperbolic or subadditive?. *Journal of Risk and Uncertainty,* 23(1), 5-32.

Regenwetter, M. & Davis-Stober, C. P. (2012). Theoretical Note: Behavioral Variability of Choices Versus Structural Inconsistency of Preferences. *Psychological Review,* 119(2), 408-416.

Regenwetter, M., Dana, J., & Davis-Stober, C. P. (2010). Testing transitivity of preferences on two-alternatives forced choice data. *Frontiers in psychology,* 1.

Regenwetter, M., Dana, J., & Davis-Stober, C. P. (2011). Transitivity of Preferences. *Psychological Review,* 118(1), 42-56.

Samuelson, P.A. (1937) A note on measurement of utility. *The Review of Economic Studies,* 4(2), 155-161.

Scholten, M., & Read, D. (2006). Discounting by Intervals: A Generalized Model of Intertemporal Choice. *Management Science,* 52(9), 1424-1436.

Scholten, M., & Read, D. (2010). The Psychology of Intertemporal Tradeoffs. *Psychological Review,* 117(3), 925-944.

Scholten, M., Read, D., & Sanborn, A. (2014). Weighting outcomes by time of against time? Evaluation rules in intertemporal choice. *Cognitive science*, 38(3), 399-438.

Tversky, A. (1969). Intransitivity of preferences. *Psychological Review,* 76, 31-48.

Urminsky, O., & Zauberman, G. (2014). *The Psychology of Intertemporal Preferences,* Wiley-Blackwell Handbook of Judgment and Decision Making.

Vincent, B. J. (2016). Hierarchical Bayesian estimation and hypothesis testing for delay discounting tasks. *Behaviour Research Methods,* 48(4), 1608–1620.

Wagenmakers, E.-J., Lee, M. D., Lodewyckx, T., & Iverson, G (2008). Bayesian versus frequentist inference. In H. Hoijtink, I. Klugkist, & P. A. Boelen (Eds.), *Bayesian Evaluation of Informative Hypotheses,* (pp. 181-207). New York: Springer Verlag.

Wagenmakers, E. J., Morey, R. D., & Lee, M. D. (2016). Bayesian benefits for the pragmatic researcher. *Current Directions in Psychological Science,* 25(3), 169-176.

Appendix A

Instructions for the time task:

You'll be presented with a series of hypothetical pairs of alternatives from which you should pick one based on your preference. Each one of the alternatives shown differs both in the amount of money being offered and the time of delivery. For example: Which alternative would you prefer? A = 300 mexican pesos in 6 weeks or B = 400 mexican pesos in 6 weeks. In this particular case, choosing the A alternative means that you would receive 300 pesos 5 weeks from now, whereas if you choose option B you would receive 400 pesos within 6 weeks from now.

The instructions to select the alternatives were the following:

To choose between the alternatives shown on screes you need to place the mouse over the corresponding letter of your preferred choice and it will change to an orange color. You must then click the mouse to indicate that you have selected this particular alternative. Once you have chosen an alternative with the click of the mouse, the screen will confirm which was the alternative that you chose. In order to continue to the next pair of alternatives, you will need to click in the center of the screen. In the next trial, you will have to choose between another pair of alternatives which will also be different in terms of the amount of money being offered and the time of delivery. There is no wrong or right answers, we are just interested in exploring your preferences. Each one of the questions presented is important, please choose carefully. If you are to begin with the experiment, click anywhere on the screen.

Appendix B

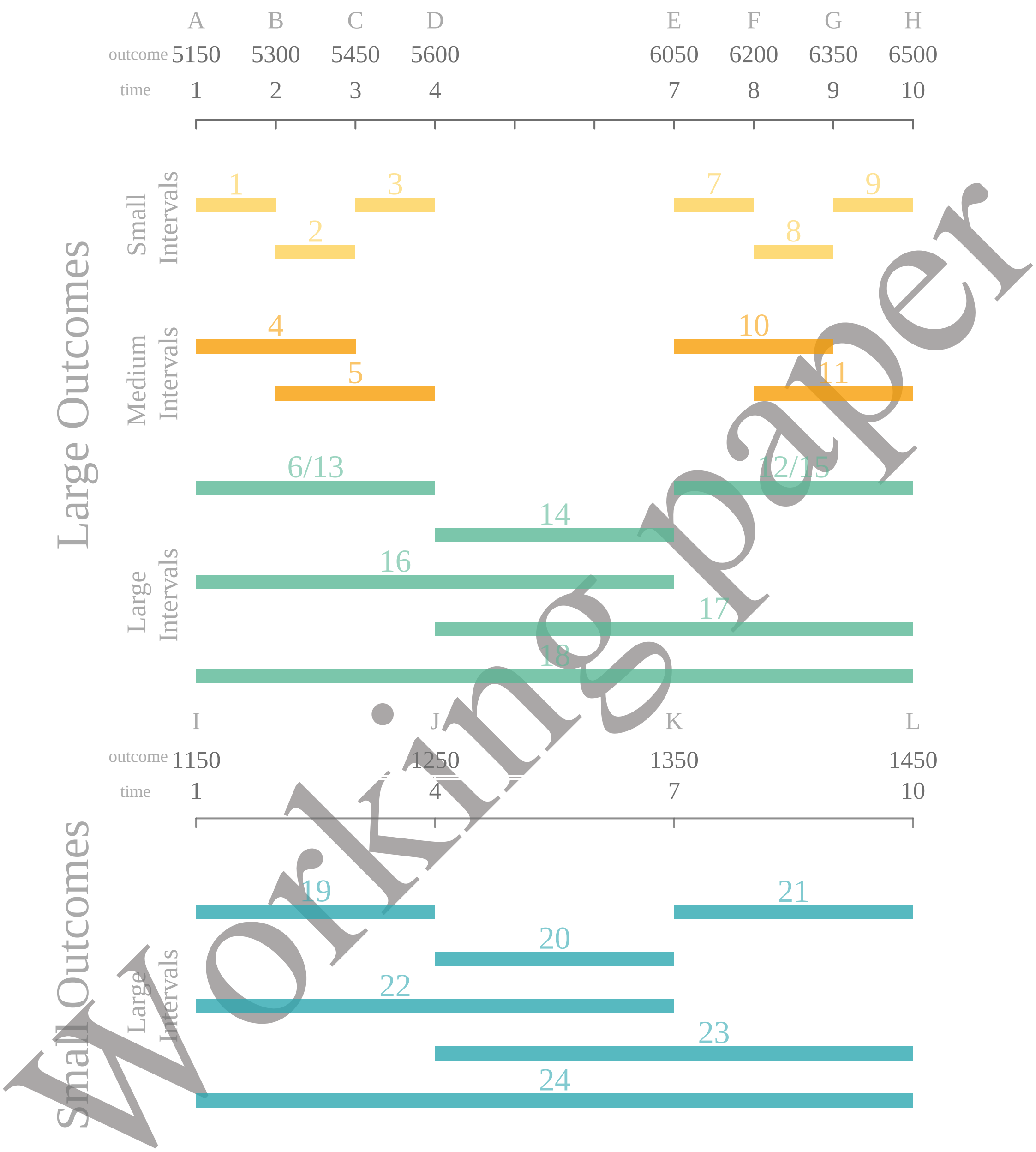

Figure 7. Graphical representation of the Experimental Design. Each bar represents a question, the leftmost end indicates the SSR and the rightmost end the LLR. A total of 22 questions were created by combining 12 alternatives, each one identified with letters from A to L, and indicating an outcome and a delay. There are four sets: 1) Small intervals, 2) Medium intervals, 3) Long intervals/Large outcomes, and 4) Long intervals/Small outcomes.

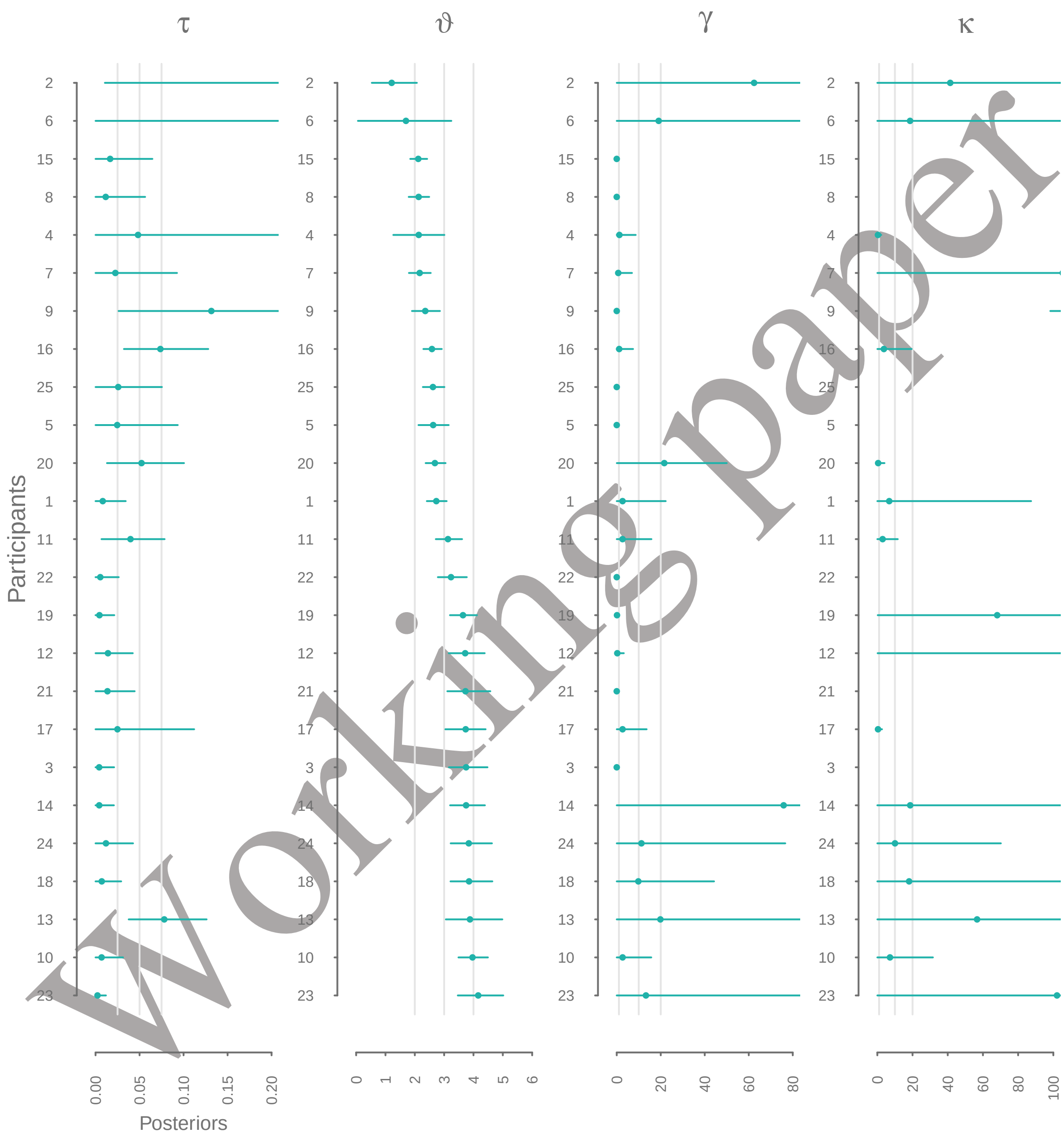

Figure 8. Individual interval posterior distributions as computed by the Trade-off model.

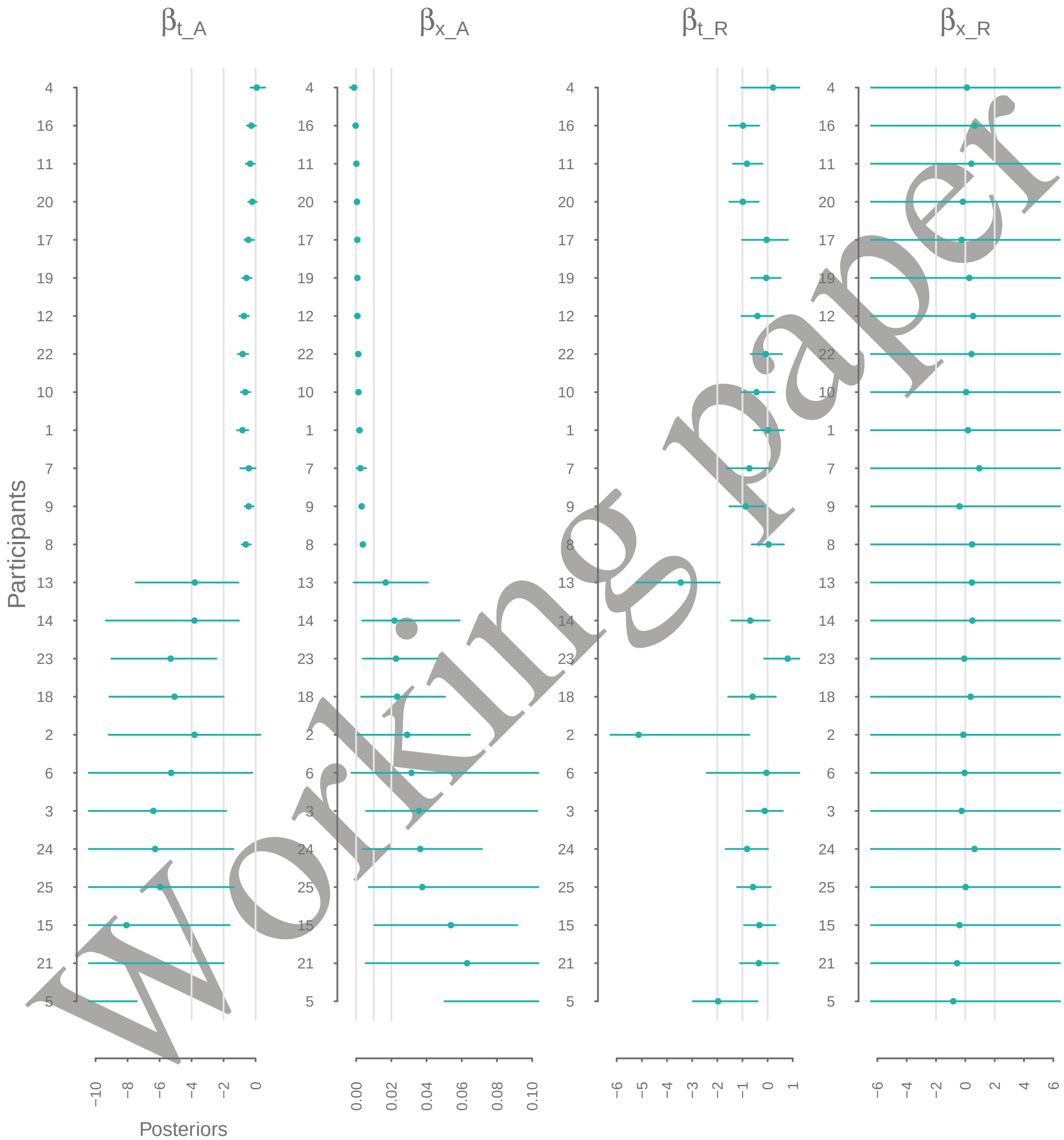

Figure 9. Individual interval posterior distributions as computed by the ITCH model.

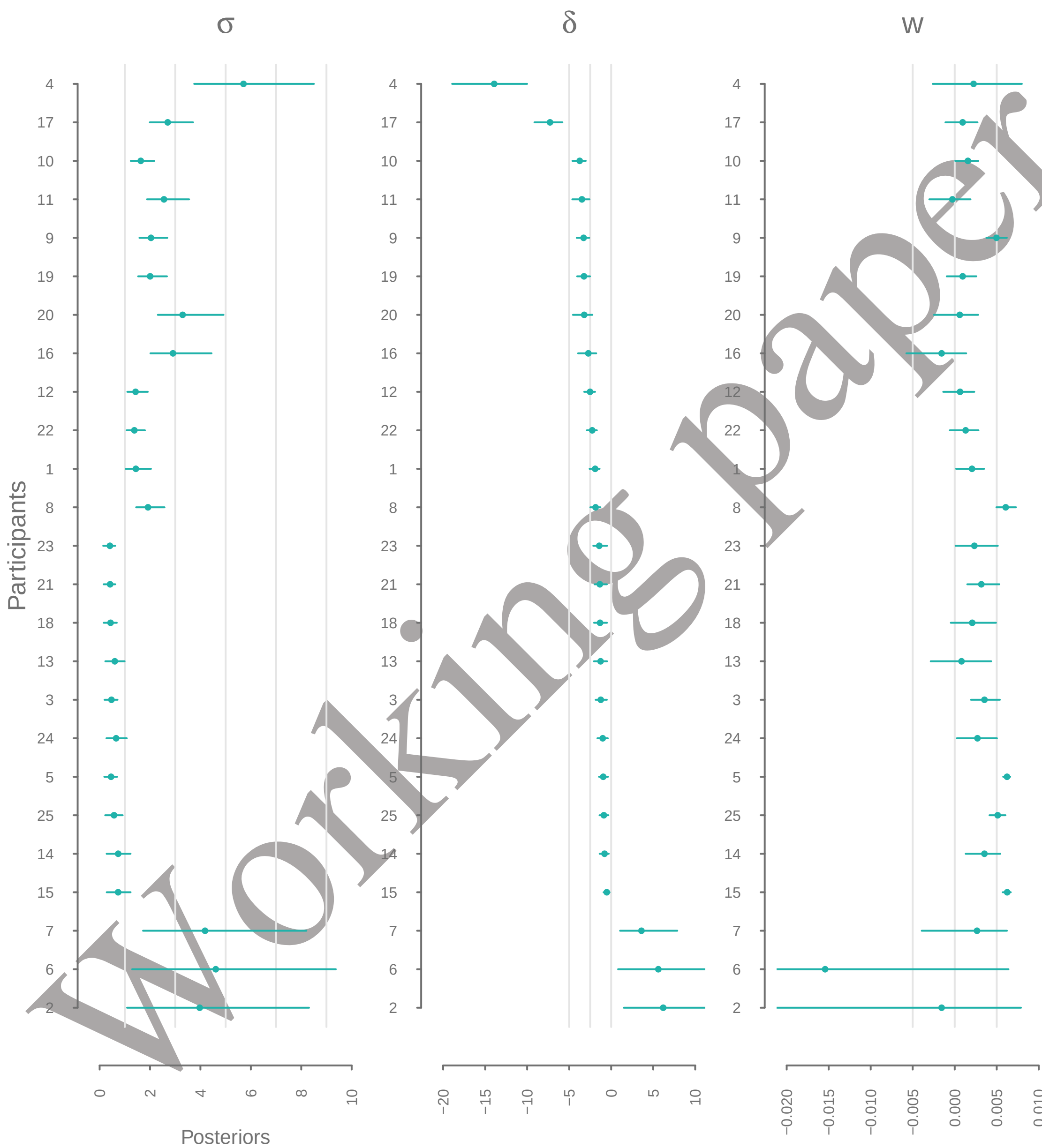

Figure 10. Individual interval posterior distributions as computed by the Direct Differences model.